Dynamic Epistemic Logic as a Method for Solving Probabilistic Paradoxes

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Abstract

There are several approaches to solving probabilistic paradoxes - problems generating cognitive distortions in the calculation of a probability measure. For example, such problems are solved by means of probability theory or game theory. However, since the difficulties are due to the imperfection of the agent's information, it seems natural to apply the methods of epistemic logic - in which probabilistic estimates and the possibility of describing information dynamics are integrated - to the solution. The purpose of this study is to prove the adequacy of the use of PDEL logic for solving probabilistic paradoxes and to indicate its advantages over classical solutions.

1. Probabilistic paradoxes

1.1 The Monty Hall problem

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

1.2 Three Prisoners problem

Three prisoners, A, B and C, are in separate cells and sentenced to death. The governor has selected one of them at random to be pardoned. The warden knows which one is pardoned, but is not allowed to tell. Prisoner A begs the warden to let him know the identity of one of the others who are going to be executed. "If B is to be pardoned, give me C's name. If C is to be pardoned, give me B's name. And if I'm to be pardoned, flip a coin to decide whether to name B or C." The warden tells A that B is to be executed. Prisoner A is pleased because he believes that his probability of surviving has gone up from 1/3 to 1/2, as it is now between him and C. Prisoner A secretly tells C the news, who is also pleased, because he reasons that A still has a chance of 1/3 to be the pardoned one, but his chance has gone up to 2/3. What is the correct answer?

2. Probabilistic solution

Within the framework of probability theory, paradoxes are solved by the Bayes theorem, which determines the probability of an event, provided that there was another event statistically interdependent with the first.

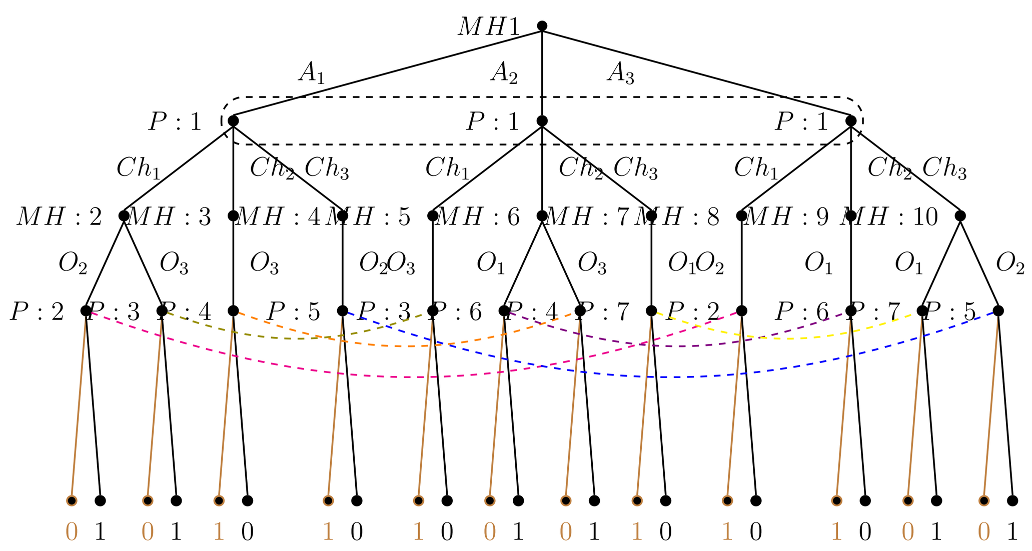
$$P(H_k|A) = \frac{P(H_k|A) \times P(H_k)}{\sum_{i=1}^{n} P(H_i|A) \times P(H_i)}$$

In the Monty Hall problem, it is necessary to calculate the probability that a goat is behind a certain door, provided that a certain door is opened. In the problem of the Three Prisoners, it is necessary to determine the probability that prisoner A will be pardoned, provided that the warder has reported that Prisoner B will be executed.

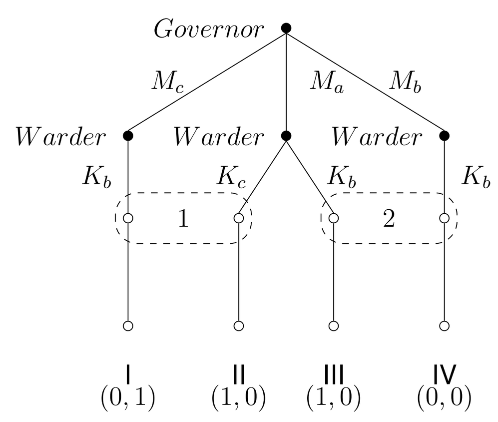
3. Game-Theoretic Solution

3.1 Solution of the Monty Hall problem

In game theory, one can analyze the problem of Monty Hall as a game with complete imperfect information. The complete game tree that combines all the variants of the development of event has an extensive graphic and matrix representation. In the full game Monty has 24 strategies, and the participant - 192. This number of strategies gives 4608 strategy profiles, which makes analysis difficult.



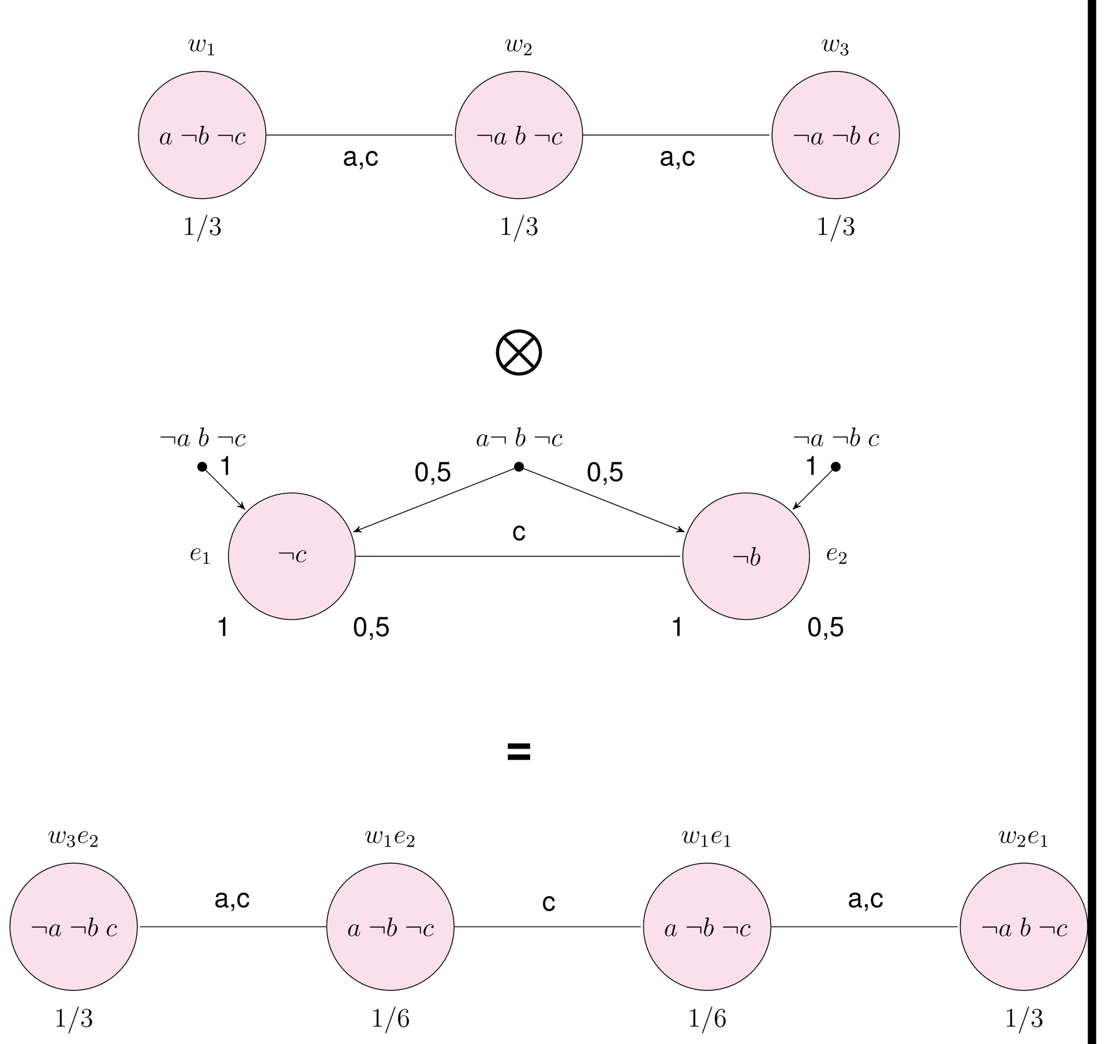
3.2 Solution of the Prisoners problem



The interaction in Three Prisoners paradox is not strategic. However, since events occur sequentially, their sequence can be represented as a tree. The task is to estimate the probability that A or C are at some particular node.

4. PDEL solution

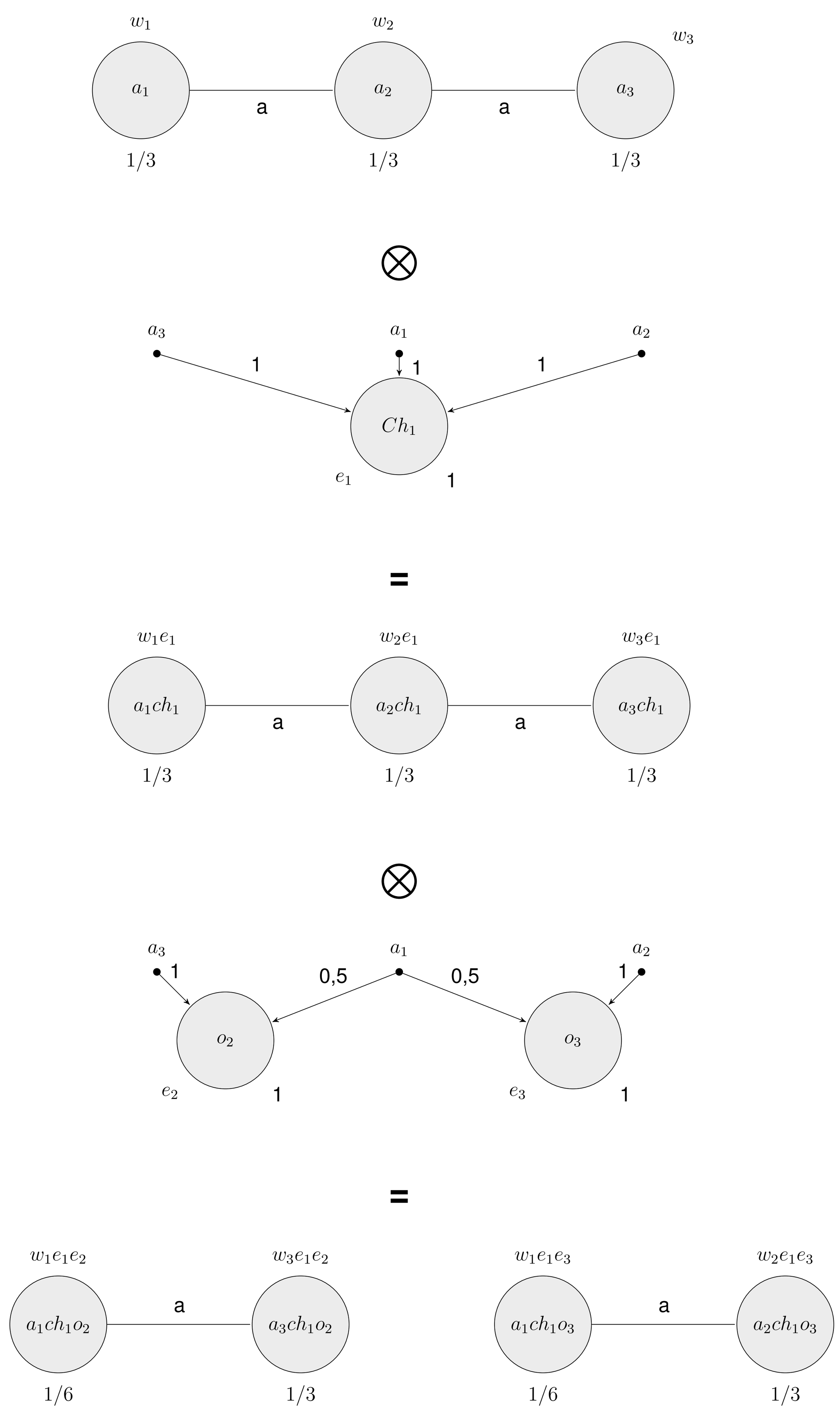
4.1 Solution of the Prisoners problem



To represent the solution of the paradox of the Three Prisoners in probabilistic dynamic epistemic logic, it is necessary to construct a model structure consisting of epistemic probability model, probabilistic update model and updated model; and then to estimate the probability in the latter. Here the epistemic probability model will reflect the probabilities of the states before the guard's report. And the probabilistic update model contains the probability of each of the information updates and their preconditions.

4.2 Solution of the Monty Hall problem

In the Monty Hall problem, the epistemic probability model reflect the probabilities of certain distribution of prizes before opening of the doors. And the probabilistic update model contains the probability that the presenter will open a certain door and conditional probabilities.



5. PDEL semantics

Aprior model: $M = \langle W, \{\sim_i\}_{i \in Ag}, \{\pi_i\}_{i \in Ag}, V \rangle$

Updating model: $E = \langle E, \{R_i\}_{i \in Ag}, \Phi, \{P_i\}_{i \in Ag}, pre \rangle$

Updated model: $M \times E = M' = \langle W', \sim', \{\pi'_i\}_{i \in Ag}, V' \rangle$

- Set: $W' = \{(w, e) \mid w \in W, e \in E, pre(w, e) > 0\}$
- Set: $\sim'_i = \{(w, e) \sim'_i (w', e') | w \sim_i w', e \sim_i e'\}$
- Function: $V'(w, e) = V(w)$
- Function:

$$\pi'_i(w, e) = \frac{\pi_i(w|[w]_i) \times pre(w, e) \times P_i(e|[e]_i)}{\sum_{w' \in W, e \in E} \pi_i(w'|[w]_i) \times pre(w', e') \times P_i(e'|[e]_i)}$$

6. Conclusion

It was found that PDEL has the advantage that it allows to work with high-order information and draws a distinction between types of probabilities. The probability distribution in this method is related to the specific situation of the paradox, where the probabilities n depend not only on the natural law, but also on the imperfection of the agent's information. It was revealed that a significant advantage is the ability to simultaneously work with the information space of several agents, as well as the possibility of a phased analysis of paradoxes.