All Properties are Divine or God Exists

The Sacred Thesis and its Ontological Argument,

with Apathiatheistic and Confidentialistic Remarks

Lecture for the the joint meeting of the Logico-philosophical club and Formal philosophy group at HSE, Moscow

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Some Historical Background

Gödel's ontological argument seems to have been inspired by ideas in Leibniz concerning the notion of 'perfection'. I do not know whether Leibniz believed an ontological argument for a perfect being was possible, but he was concerned with showing that the notion of a perfect being is *coherent*.

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Since the late eighties many authors analysed Gödel's argument, and we can talk more about these below and during discussion.

Gödel's and Anderson's accounts

DG1 $Gx \triangleq \forall I(\mathfrak{D}I \rightarrow Ix)$

DA1 $Gx \triangleq \forall I(\mathfrak{D}I \leftrightarrow \Box Ix)$

AA1 $\mathfrak{D}A \rightarrow \neg\mathfrak{D}\bar{A}$

AA2 $\mathfrak{D}A \wedge \Box\forall x(Ax \rightarrow Bx) \rightarrow \mathfrak{D}B$

AA3 $\mathfrak{D}G$

AA4 $\mathfrak{D}A \rightarrow \Box\mathfrak{D}A$

In our idiolect Gödel presupposed an axiom **AG1** stating that $\mathfrak{D}A \leftrightarrow \neg\mathfrak{D}\bar{A}$ instead of Anderson's **AA1**, and the translation of Gödel's **AG2** to **AG4** would be as **AA2** to **AA4**. Anderson and Gödel further presupposed axiomatically that some defined property akin to *necessary existence* is divine, and below we refer to these respective assumptions as **AA5** and **AG5**.

Sobel's challenge and its rectificarions

Sobel 1987 showed that Gödel's apparatus ledas to modal collapse so that $p \rightarrow \Box p$ becomes a theorem.

Anderson 1990 succeded in proposing an amended argument which avoided Sobel's collapse of modalities.

Later Hájek 1996 and 2002 showed that Gödel's argument avoids modal collapse with a weakening of second order comprehension.

Both Hájek and Anderson presupposed second order B.

Hájek's moves

In 1996 and 2002 Hájek improves upon a just slightly insufficient model theoretic argument by Magari in 1988 to the effect that Gödel's **AG1**, **AG2** and **AG3** already suffice for the main theorem. Magari's idea is confirmed as applied to Anderson's argument for Hájek shows that Anderson's **AA3** and **AA4** are superfluous in the presence of full second order modal comprehension with second order B as the underlying logic. Hájek also shows that such a comprehensive Andersonian argument is interpretable in Gödel's original set of axioms with a *cautious* comprehension principle, and that such a cautious version of Gödel's argument does not lead to the modal collapse which Sobel derived presupposing full comprehension in 1987.

The LOGICA Yearbook 1998 Result

The result of my LOGICA Yearbook 1998 article is that Gödel's AG2, AG3 and AG4 are superfluous in an argument equivalent under second order S4 and with an amended axiom for the positivity of necessary existence. Here the predicate G is taken as fundamental (see below).

The Project

Our approach below develops an argument of a manuscript circulated some under the title
If Some Property is not Divine then God Exists from 1998 which made it so dated to the discussion and bibliography of (Fuhrmann 2005). The result of said manuscript was indicated at the end of (Bjørdal 1999):

The LOGICA-Yearbook 1998 statement of the result

"By making use of a result of Petr Hajek (see (Hajek1996)), which he made me aware of at the Liblice-conference, and presupposing certain recursive definition-clauses for divine (positive) and Godly being, we may show that even Ax. 2 is eliminable if we presuppose a reasonable second order comprehension principle for the predicate Godly being.... I hope to be able to publish this improved result, alongside with certain remarks, in a future paper."

Taking **Godly being** as primitive

A notable difference between Kurt Gödel's argument and the one I offered in (Bjørdal 1999) is that Gödel takes the second order property *positive property* (we prefer *divine property*) as primitive wheras I take the first order property *Godly being* as primitive. This difference is of central importance in simplifying many matters.

Renewed interest

Recently the author's work and (Bjørdal1999) received favorable attention from Christoph Benzmüller and Bruno Woltzenlogel-Paleo; on this see link from Benzmüller's home page.

An unpublished manuscript had evolved and I recently found version (Bjørdal 2011) which with other work is superseded by considerations below and forthcoming.

Eliminating apparent circularity with higher order

As pointed out e.g. in (Belnap & Gupta 1993) p. 194, seemingly circular definitions may be appropriately inductive and circularity (though not impredicativity, of course) avoided by higher order machinery; the particular definitional scheme referred to loc.cit. may as verified in (Gupta 2012) be simplified so that if H occurs positively in A(x,H) we can define Jx by $\forall K(\forall y(A(y,K) \rightarrow Ky) \rightarrow Kx))$ and show that $\forall x(Jx \leftrightarrow A(x,J))$ under standard assumptions.

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In our modal logical context we need a slightly different approach, and we may point out differences as we lay things out.

$\Pi^{1,2}_{m,n}$-comprehension

For third order logic, take $\Pi^{1,2}_{m,n}$-comprehension to be second order Π^1_m-comprehension plus third order Π^2_n-comprehension.

Der1

Use $\Pi^{1,2}_{1,1}$-comprehension to define, in third order modal logic:

(1) $\exists\mathfrak{D}\forall H(\mathfrak{D}H \leftrightarrow \forall\mathfrak{C}((\Box\forall x(\forall L(\mathfrak{C}L \rightarrow \Box Lx) \rightarrow Hx) \rightarrow \Box\mathfrak{C}H) \rightarrow \mathfrak{C}H))$

We existentially instantiate with a homographic letter, and it follows that

(2) $\forall H\forall\mathfrak{C}((\Box\forall x(\forall L(\mathfrak{C}L \rightarrow \Box Lx) \rightarrow Hx) \rightarrow \Box\mathfrak{C}H) \rightarrow (\mathfrak{D}H \rightarrow \mathfrak{C}H))$

It is a thesis on account of the 5-axiom and the 4-axiom that

(3) $\forall H\forall\mathfrak{C}(\Box(\Box\forall x(\forall L(\mathfrak{C}L \rightarrow \Box Lx) \rightarrow Hx) \rightarrow \Box\mathfrak{C}H))$ is strictly entailed by

(4) $\forall H\forall\mathfrak{C}(\Box\forall x(\forall L(\mathfrak{C}L \rightarrow \Box Lx) \rightarrow Hx) \rightarrow \Box\mathfrak{C}H)$.

So the following is a thesis of third order S5:

(5) $\forall H\forall\mathfrak{C}(\Box((\Box\forall x(\forall L(\mathfrak{C}L \rightarrow \Box Lx) \rightarrow Hx) \rightarrow \Box\mathfrak{C}H)) \rightarrow (\mathfrak{D}H \rightarrow \mathfrak{C}H))$

Der2

By necessitation and the second and third order converse Barcan formulas:

(6) $\forall H \forall \mathfrak{C} \Box(\Box((\Box\forall x(\forall L(\mathfrak{C}L \to \Box Lx) \to Hx) \to \Box\mathfrak{C}H)) \to (\mathfrak{D}H \to \mathfrak{C}H))$

By the K-principle and repetition it follows that

(7) $\forall H \forall \mathfrak{C}((\Box\forall x(\forall L(\mathfrak{C}L \to \Box Lx) \to Hx) \to \Box\mathfrak{C}H) \to \Box(\mathfrak{D}H \to \mathfrak{C}H))$

By the modal operativity of $\Box\forall x(\forall L(\mathfrak{C}L \to \Box Lx) \to Hx)$ in $\mathfrak{C}$ it follows that:

(8) $\forall H \forall \mathfrak{C}((\Box\forall x(\forall L(\mathfrak{C}L \to \Box Lx) \to Hx) \to \Box\mathfrak{C}H) \to$
$(\Box\forall x(\forall L(\mathfrak{D}L \to \Box Lx) \to Hx) \to \Box\forall x(\forall L(\mathfrak{C}L \to \Box Lx) \to Hx)))$

Der3

By a truth functional argument on (8):

$$(9)\ \forall H \forall \mathfrak{C}((\Box \forall x(\forall L(\mathfrak{C}L \to \Box Lx) \to Hx) \to \Box \mathfrak{C}H) \to (\Box \forall x(\forall L(\mathfrak{D}L \to \Box Lx) \to Hx) \to \Box \mathfrak{C}H))$$

By instantiation with H and change of order:

$$(10)\ \Box \forall x(\forall L(\mathfrak{D}L \to \Box Lx) \to Hx) \to \forall \mathfrak{C}((\Box \forall x(\forall L(\mathfrak{C}L \to \Box Lx) \to Hx) \to \Box \mathfrak{C}H) \to \Box \mathfrak{C}H))$$

Der4

By the T-axiom:

(11) $(\Box\forall x(\forall L(\mathfrak{D}L \to \Box Lx) \to Hx) \to$
$\forall\mathfrak{C}((\Box\forall x(\forall L(\mathfrak{C}L \to \Box Lx) \to Hx) \to \Box\mathfrak{C}H) \to \mathfrak{C}H))$

By definition of $\mathfrak{D}$H in (1):

(12) $\Box\forall x(\forall L(\mathfrak{D}L \to \Box Lx) \to Hx) \to \mathfrak{D}H$

Define:

(13) $\mathfrak{F}H \triangleq \Box\forall x(\forall L(\mathfrak{D}L \to \Box Lx) \to Hx)$

We have derived the thesis:

(14) $\Box\forall H(\mathfrak{F}H \to \mathfrak{D}H)$

Der5

Again, by the modal operativity of $\Box\forall x(\forall L(\mathfrak{D}L \to \Box Lx) \to Hx)$ in $\mathfrak{D}$ it follows that:

(15) $\Box\forall x(\forall L(\mathfrak{F}L \to \Box Lx) \to Hx) \to \Box\forall x(\forall L(\mathfrak{D}L \to \Box Lx) \to Hx)$

By the definition of $\mathfrak{F}H$ in (14) we then have:

(16) $\Box\forall x(\forall L(\mathfrak{F}L \to \Box Lx) \to Hx) \to \mathfrak{F}H$

By the 4-axiom of third order S5 we have:

(17) $\Box\forall x(\forall L(\mathfrak{F}L \to \Box Lx) \to Hx) \to \Box\mathfrak{F}H$

Der6

By invoking the definition of $\mathfrak{D}H$ in (1) we then conclude that

(18) $\Box\forall H(\mathfrak{D}H \rightarrow \mathfrak{F}H)$

Combining, we have:

(19) $\Box\forall H(\mathfrak{D}H \leftrightarrow \mathfrak{F}H)$

This is to say:

(20) $\Box\forall H(\mathfrak{D}H \leftrightarrow \Box\forall x(\forall L(\mathfrak{D}L \rightarrow \Box Lx) \rightarrow Hx))$

Der7

Define, by using Π^1_1-comprehension:

(21) $\Box\forall x(Gx \leftrightarrow \forall L(\mathfrak{D}L \rightarrow \Box Lx))$

From (20) and (21),

(22) $\Box\forall H(\mathfrak{D}H \leftrightarrow \Box\forall x(Gx \rightarrow Hx))$

(20-22) are the apparently circular second order conditions justified by the third order definition (1); in complexity these involve just the non-circular impredicative $\Pi^{1,2}_{1,1}$-comprehension on modalized conditions as per above.

Der8

From (22) and the fact that $\Box\forall x(Gx \rightarrow Gx))$ is a thesis:

(23) $\mathfrak{D}$G

By the 4-axiom:

(24) $\Box\mathfrak{D}G$

From (22) and the fact that $\Diamond\exists x(Gx \wedge \neg Xx) \rightarrow \Diamond\exists x(Gx)$

(25) $\neg\mathfrak{D}X \rightarrow \Diamond\exists x(Gx)$

From (21) by instantiation, simplification and permutation:

(26) $\Box(\mathfrak{D}G \rightarrow \forall x(Gx \rightarrow \Box Gx))$

Der8

From (24) and (26):

(27) $\Box\forall x(Gx \rightarrow \Box Gx)$

From (27):

(28) $\Box(\exists xGx \rightarrow \exists x\Box Gx)$

The following is a theorem of all quantified modal logics:

(29) $\Box(\exists x\Box Gx \rightarrow \Box\exists xGx)$

From (28) and (29):

(30) $\Box(\exists xGx \rightarrow \Box\exists xGx)$

Der8

From (30) and the 4-axiom:

(31) $\Box(\exists xGx \rightarrow \Box\Box\exists xGx)$

From (31) using the *K*-principle:

(32) $\Diamond\exists xGx \rightarrow \Diamond\Box\Box\exists xGx$

From the *Brouwer*-schema:

(33) $\Diamond\Box\Box\exists xGx \rightarrow \Box\exists xGx$

Der8

From (32) and (33):

(34) $\Diamond\exists xGx \rightarrow \Box\exists xGx$

Repeting line (25):

(35) $\neg\mathfrak{D}X \rightarrow \Diamond\exists x(Gx)$

From line (34) and (35):

(36) $\neg\mathfrak{D}X \rightarrow \Box\exists x(Gx)$

As X is arbitrary, by generalization:

(37) $\forall X(\neg\mathfrak{D}X \rightarrow \Box\exists x(Gx))$

Deriving the Divine Thesis

From the quantifier rules:

(38) $\exists X \neg \mathfrak{D} X \rightarrow \Box \exists x (Gx))$

By interdefinability:

(38) $\forall X \mathfrak{D} X \vee \Box \exists x (Gx)$ **The Sacred Thesis**

All properties are Divine or necessarily God exists.

From the Divine Thesis to an Ontological Argument

One may use the negation of the first disjunct of the Divine Thesis as premise in an ontological argument for the existence of a God, so the Divine Thesis supports the *validity* of:

OA: *God necessarily exists, because some property is not divine.*

A Fixed Point Analysis

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A Taming of Gaunilo-like Objections

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Nevertheless, there is a similar *diabolical thesis* and related theses for other second order properties and corresponding bearers. However, such logical theses as the Divine Thesis or the diabolical thesis do not by themselves carry ontological commitments, and they are tame by attitudes such as apathiatheism in the following.

Apathiatheism and Confidentialism

An *apathiatheistic* remark is that the best concepts of ‘God’ are such that the question as to whether there is a God or not is academic in a sense similar to the question as to whether there are holes or just holed things.

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In my apathiatheistic opinion the most important religious question is not whether there is a God, but whether something ultimately rectifies the unsayable sufferings of some (and others, for metaphysical parity), or not; my *confidentialistic* remark is that that question has an affirmative answer.

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To my mind, apathiatheism and confidentialism are compatible with reasonable interpretations of orthodox Christianity as well as with reasonable interpretations of atheism.

Last pre-bibliographic slide

Thank you for your attention!

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