НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ

Epistemic Taxonomy of Assertions

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Plan

- Types of Assertions and Some Theoretical Difficulties
- Elements of Epistemic/Doxatic Logic
- Strong Common Belief
- An Epistemic Taxonomy of Assertions

TYPES OF ASSERTIONS

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5. 'You are the cream in my coffee'
6. 'Sky is blue'

Theoretical Difficulties

Maxim of Quality:
Do not say what you believe to be false.
Do not say that for which you lack adequate evidence.

- Flouting of the Maxim of Quality
- irony, metaphor, etc
- deception vs. irony

Maxim of Quantity

Maxim of Quantity: Be as informative as required.

- 'Boys are Boys'
- 'A is A'

Epistemic Presuppositions

S: φ to H at c.
Epistemic presuppositions describe S's and H's higher-order epistemic and doxatic attitudes (φ, c).

Meta-knowledge and implicature

know vs. think
«...the speaker thinks and (would expect the hearer to think that the speaker thinks)...» [Grice 1989, p.31]
«...he (and knows that I know that he knows...» [Grice 1989, p.31]

Types of Presuppositions

Semantic presupposition:
should be associated with specific triggers ('stop', 'continue', 'regret', ...)
Pragmatic presupposition:
«A speaker presupposes that P at a given moment in a conversation just in case he is disposed to act, in his linguistic behavior, as ifhe takes the truth ofP for granted, and as ifhe assumes that his audience recognizes that he is doing so».
[Stalnaker 1975, p.32]

ELEMENTS OF EPISTEMIC/DOXATIC LOGIC

The syntax of language $\mathcal{L}_E$ is given by the following formula
$\varphi := p \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi \mid \neg\varphi \mid B_i\varphi \mid K_i\varphi$
$K_i\varphi$ - 'an agent i knows that φ'
$B_i\varphi$ - 'an agent i believes that φ'

An epistemic/doxatic model

$\mathcal{M} = \langle \mathcal{A}, W, \{\sim_i\}_{i \in \mathcal{A}}, \{\preceq_i\}_{i \in \mathcal{A}}, V \rangle$, where

- $\mathcal{A}$ – set of agents

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- $\sim_i$ – relation on W for i
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- $V : Var\mathcal{L}_E \rightarrow \mathcal{P}(W)$

Properties of $\sim_i$

- $K_i\varphi \to \varphi$

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Properties

- $K_i\varphi \to \varphi$ (Factivity)
- $K_i\varphi \to K_iK_i\varphi$ (Positive Introspection)
- $\neg K_i\varphi \to K_i\neg K_i\varphi$ (Negative Introspection)
- $B_i\varphi \to B_iB_i\varphi$ (Positive Introspection)
- $\neg B_i\varphi \to B_i\neg B_i\varphi$ (Negative Introspection)

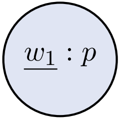
Truth in a Model

φ is true at state w in a model $\mathcal{M}$ is defined by induction

- $\mathcal{M}, w \models p$ iff $w \in V(p)$
- $\mathcal{M}, w \models \neg\varphi$ iff $\mathcal{M}, w \not\models \varphi$
- $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models \varphi \vee \psi$ iff $\mathcal{M}, w \models \varphi$ or $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models \varphi \rightarrow \psi$ iff $\mathcal{M}, w \not\models \varphi$ or $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models K_i\varphi$ iff $\forall w'(w \sim_i w' \rightarrow \mathcal{M}, w' \models \varphi)$
- $\mathcal{M}, w \models B_i\varphi$ iff $\forall w'(w' \in max_{\preceq_i}([w]_i) \rightarrow \mathcal{M}, w' \models \varphi)$
- $max_{\preceq_i}(X) := \{w \in X \mid \forall w' \in X : w' \preceq_i w\}$, where $X \subseteq W$
- $[w]_i := \{w' \in W \mid w \sim_i w'\}$

a knows p

$$\mathcal{M}_1, w_1 \models K_a p$$



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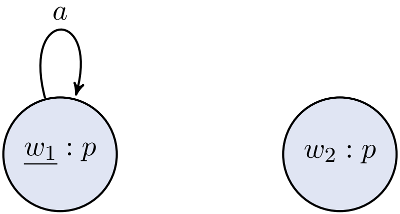

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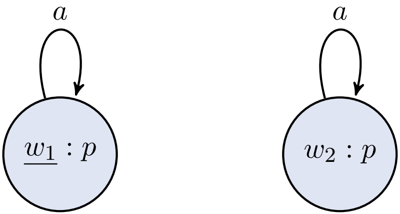

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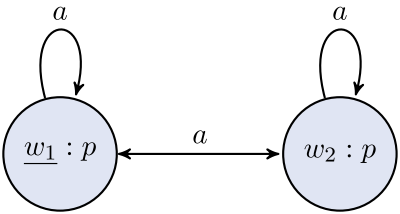

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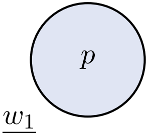

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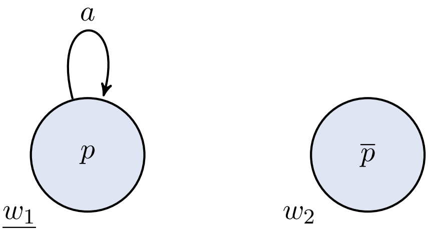

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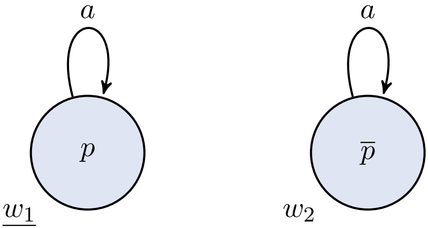

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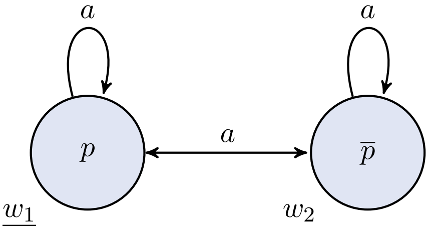

Pragmatics of question

a asks b: p?

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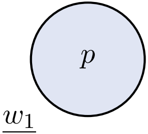

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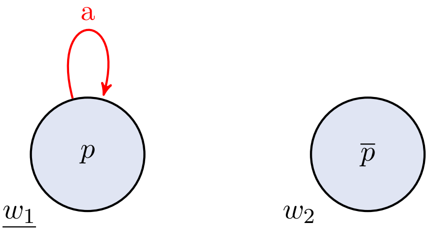

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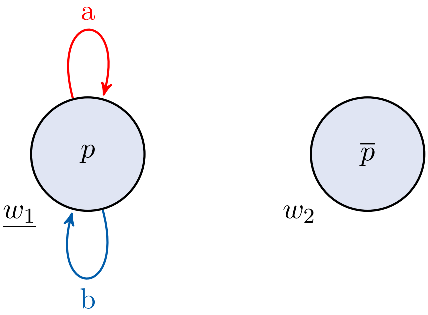

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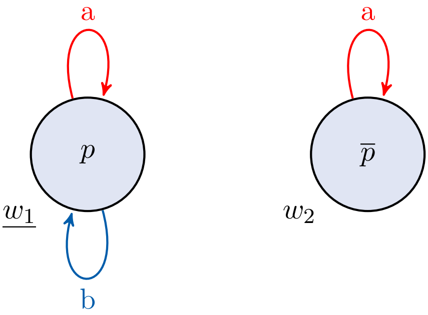

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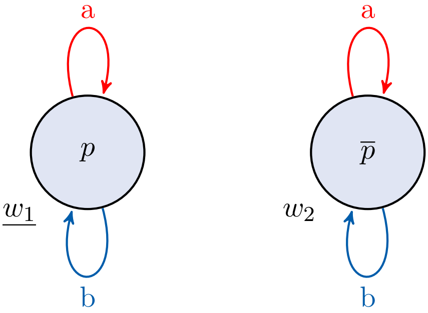

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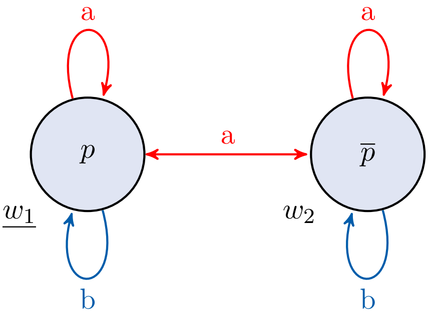
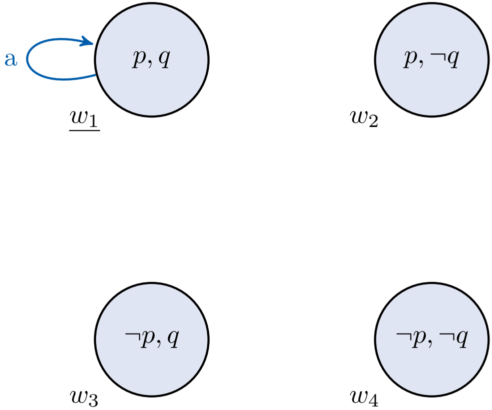
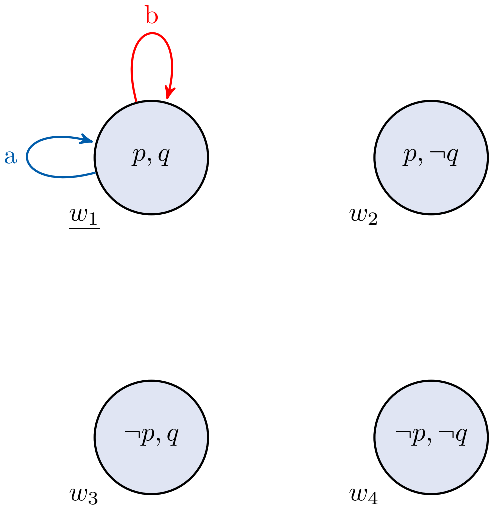



a
p, q
$\underline{w_1}$
$p, \neg q$
w_2
$\neg p, q$
w_3
$\neg p, \neg q$
w_4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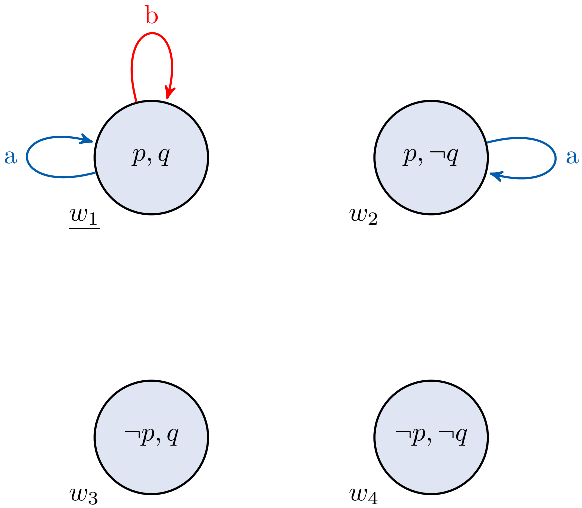
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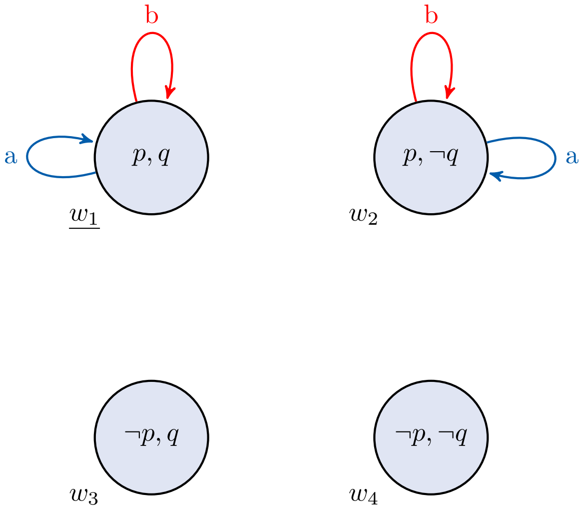
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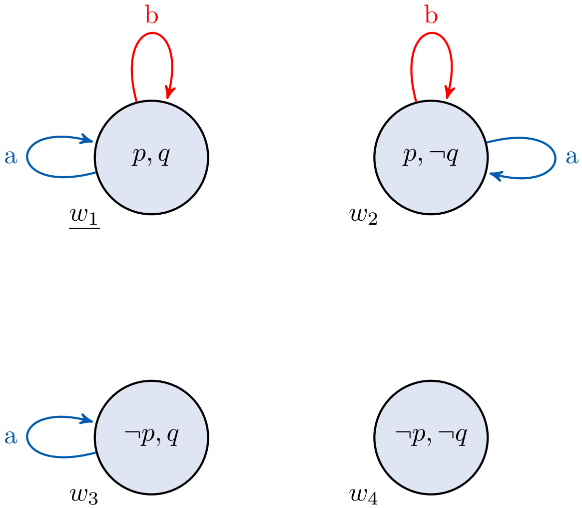
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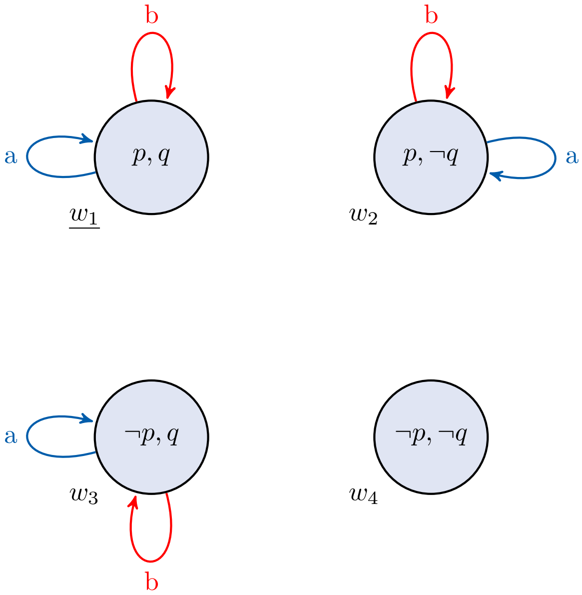

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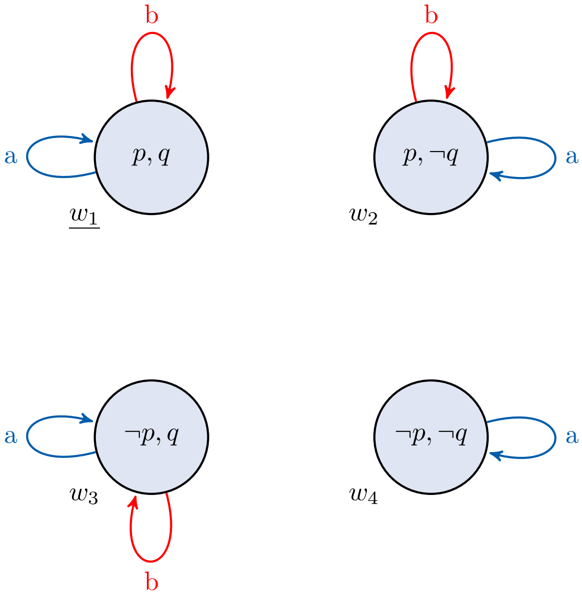

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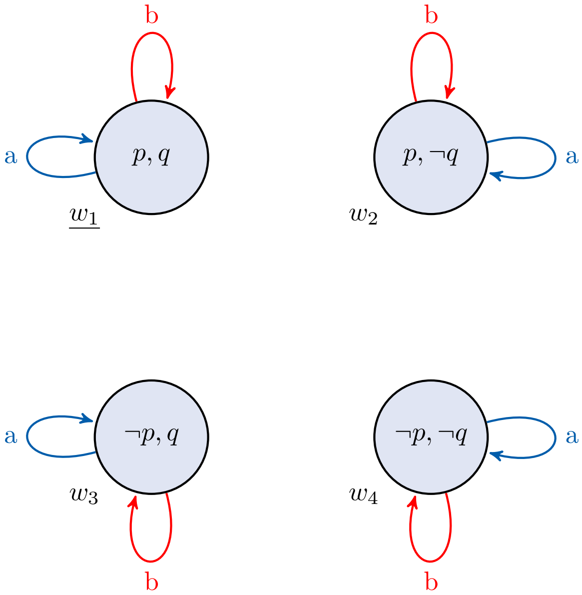

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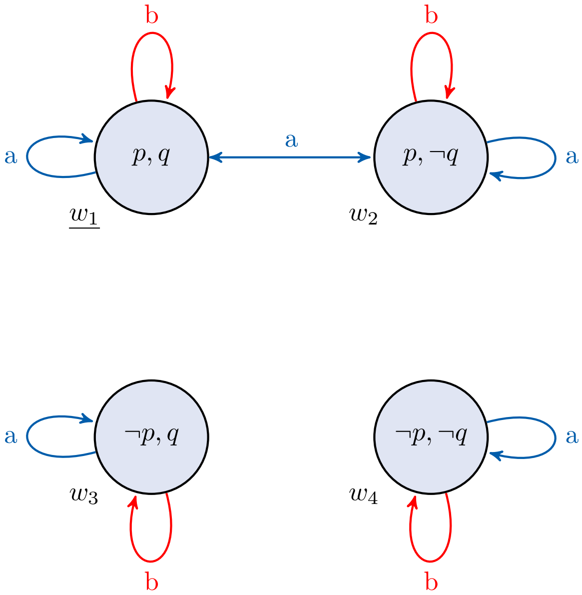
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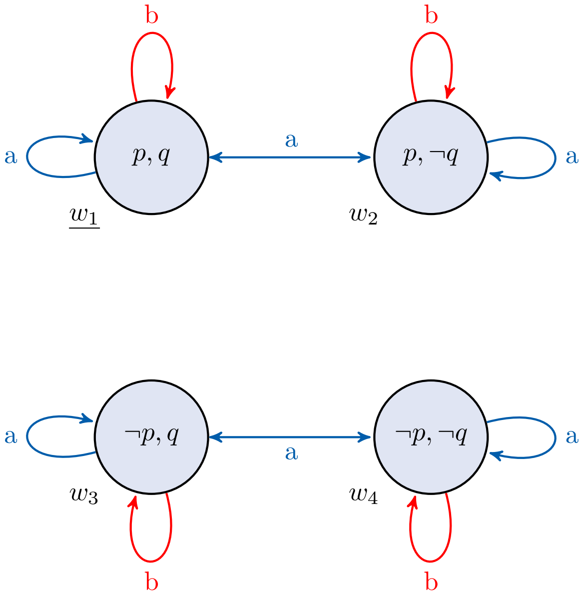
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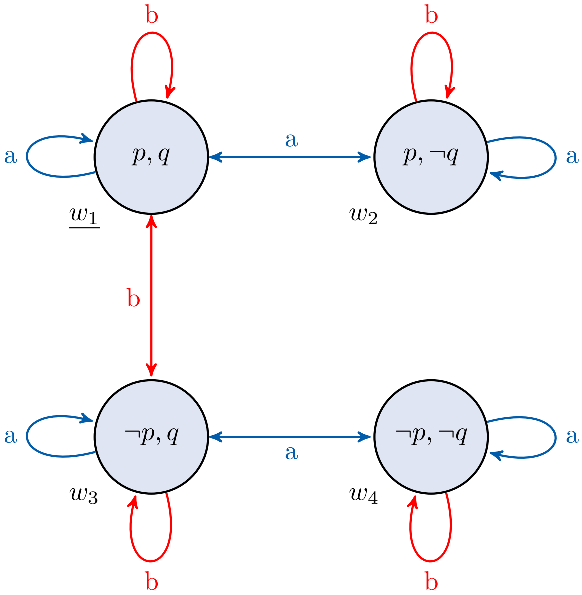

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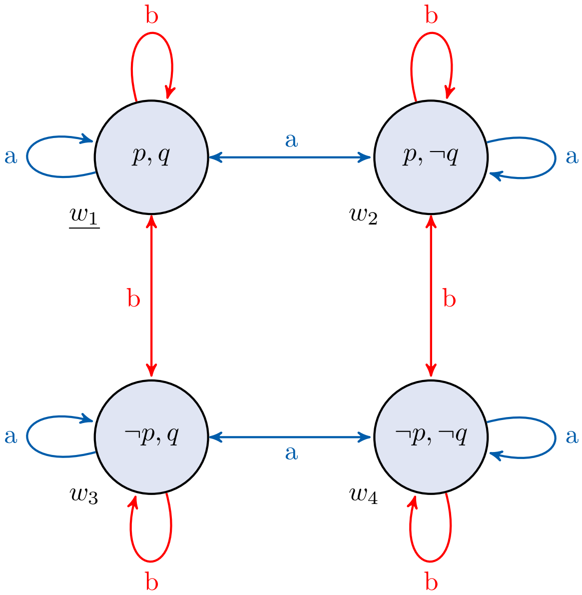
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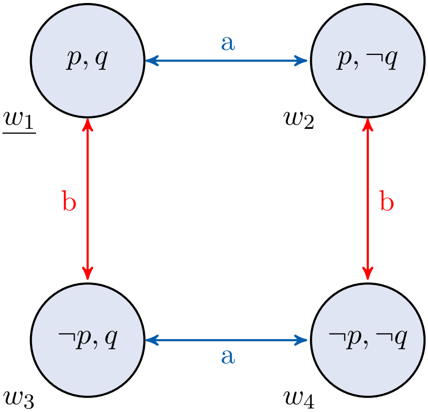
p, q
a
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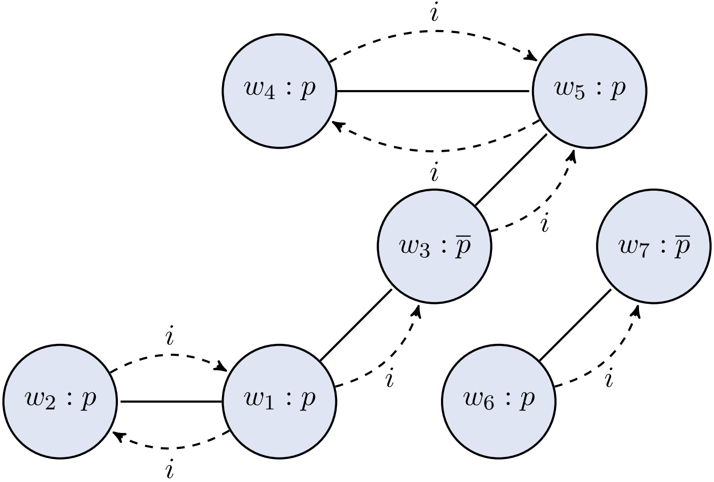
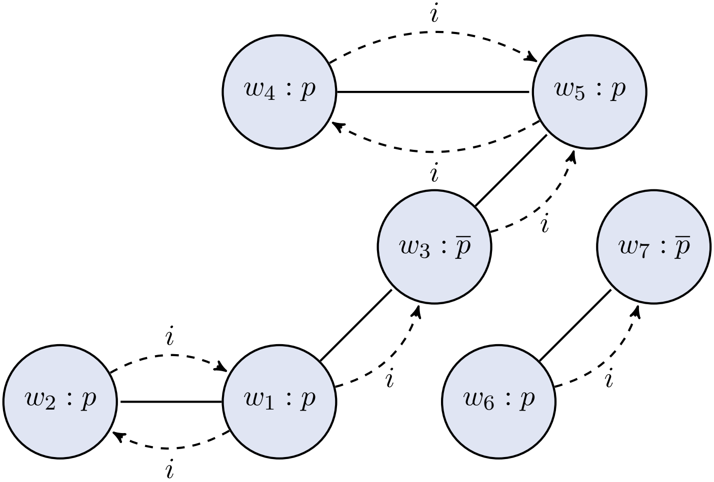


$[\varphi]_{\mathcal{M}} \leftrightharpoons \{w \in W \mid \mathcal{M}, w \models \varphi\}$, $[p]_{\mathcal{M}} = ?$



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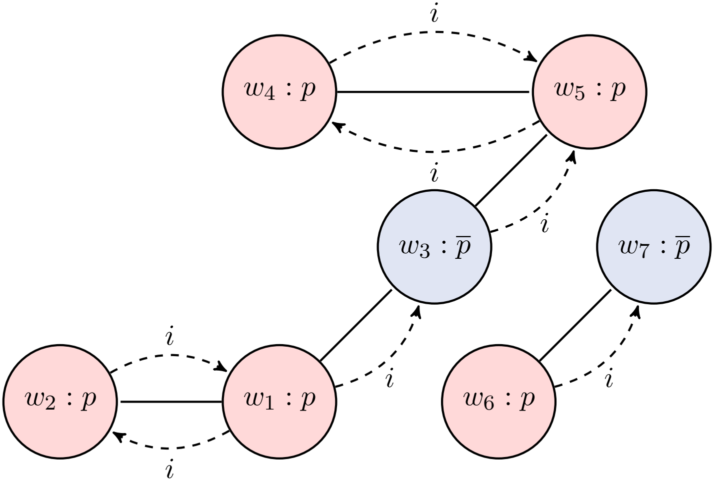

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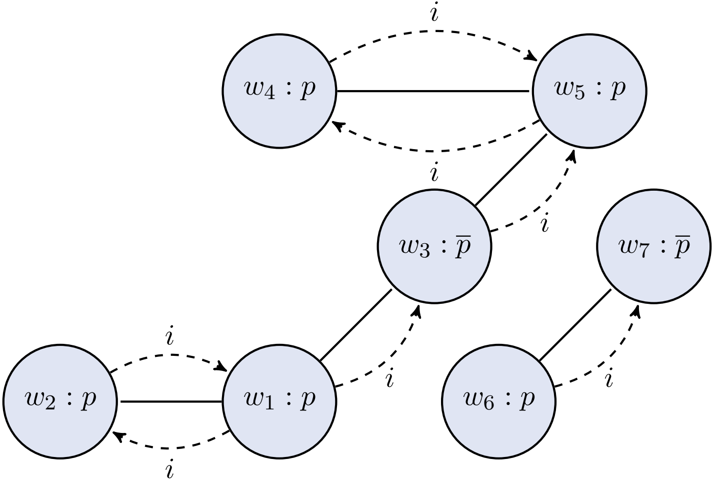

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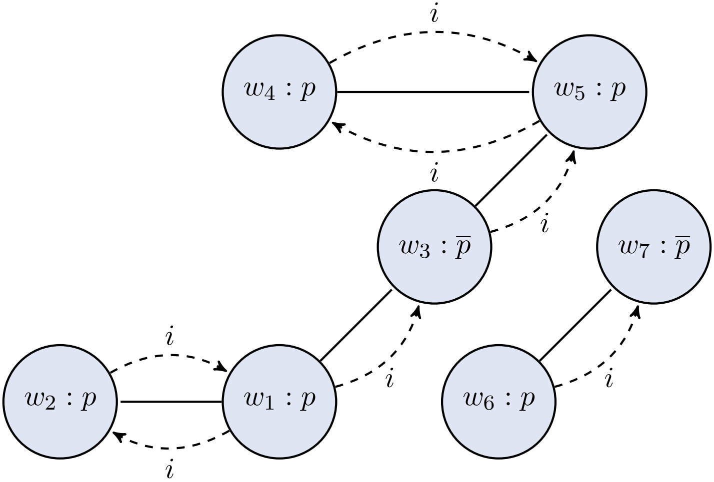

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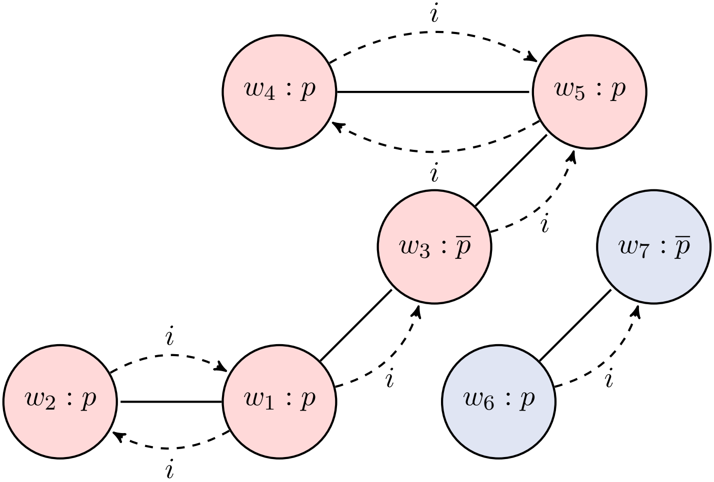

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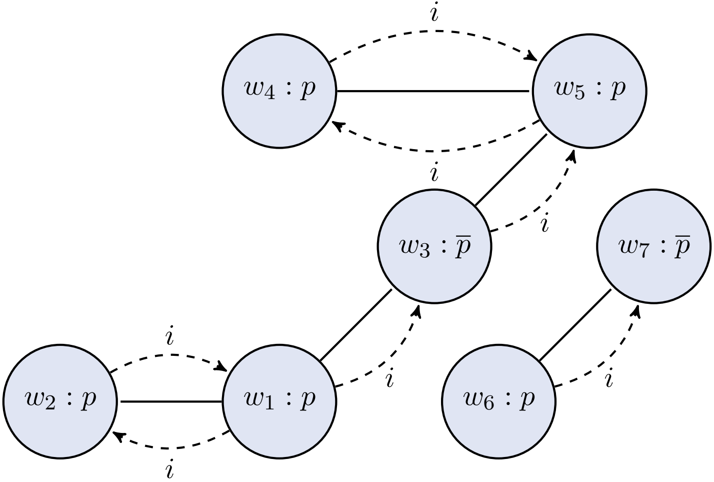

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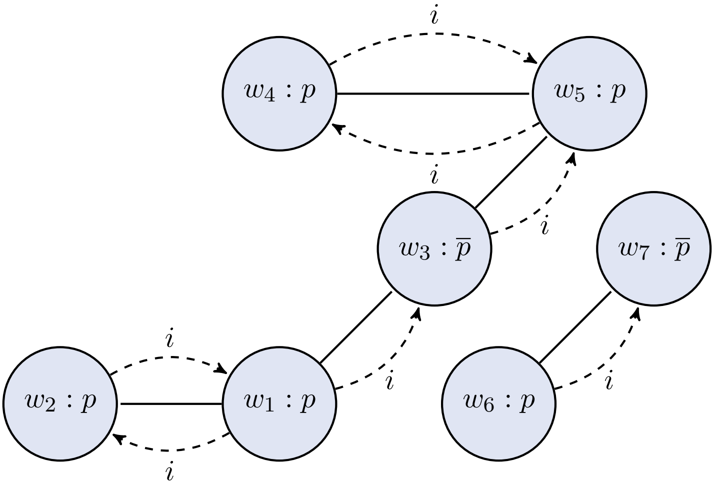
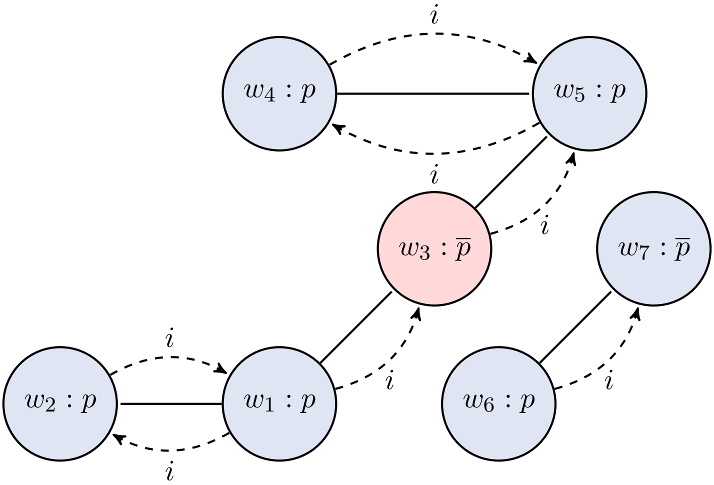


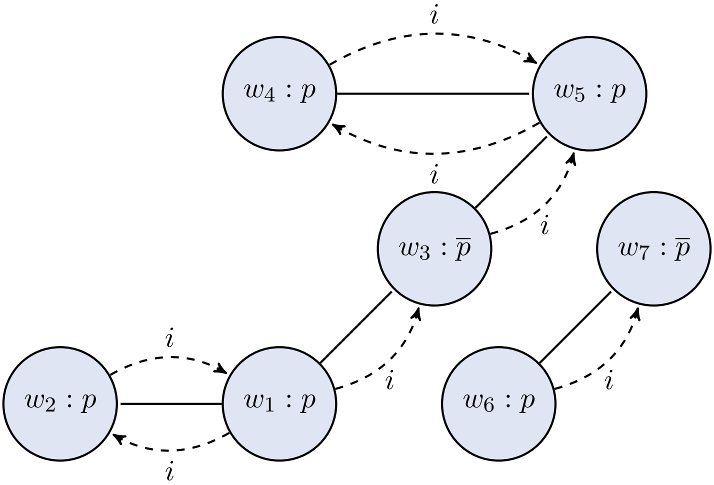
$max_{\preceq_i}(X) \leftrightharpoons \{w \in X \mid \forall w' \in X : w' \preceq_i w\}$, где $X \subseteq W$,

$max_{\preceq_i}(\{w_1, w_2, w_3\}) = \{w_3\}$



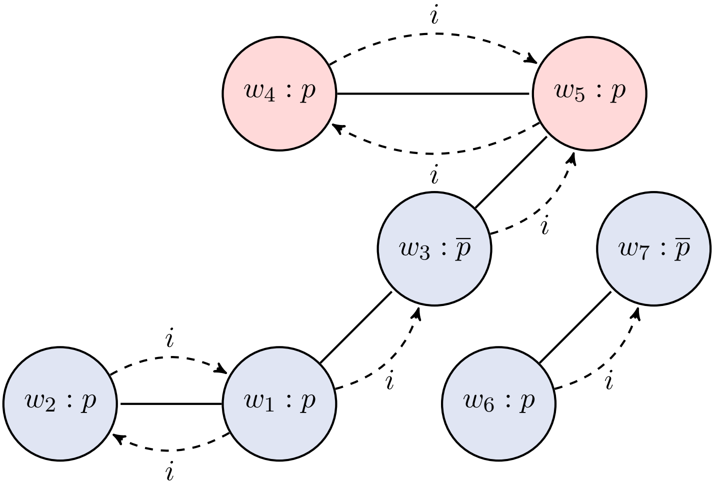
$max_{\preceq_i}(X) \leftrightharpoons \{w \in X \mid \forall w' \in X : w' \preceq_i w\}$, где $X \subseteq W$,

$max_{\preceq_i}([w_2]_i) = ?$



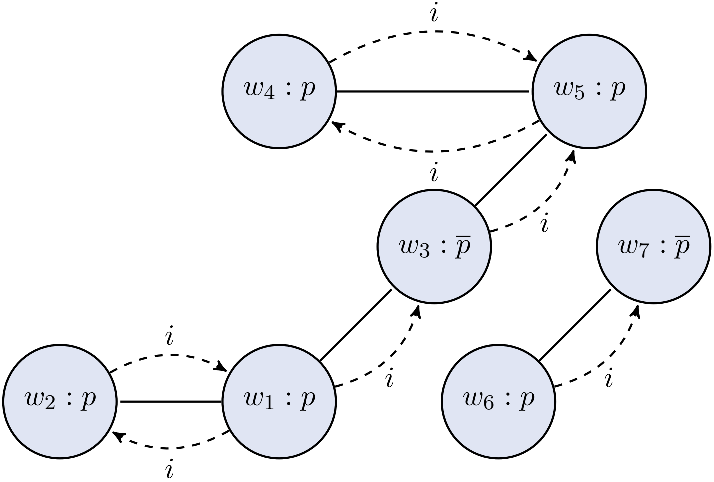
$max_{\preceq_i}(X) \leftrightharpoons \{w \in X \mid \forall w' \in X : w' \preceq_i w\}$, где $X \subseteq W$,

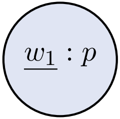
$max_{\preceq_i}([w_2]_i) = \{w_4, w_5\}$



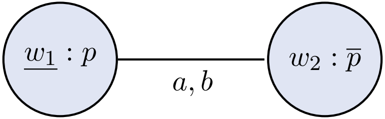
$\mathcal{M}, w_1 \models B_i p$

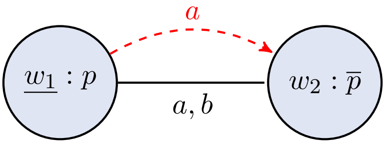
$\mathcal{M}, w_6 \not\models B_i p$



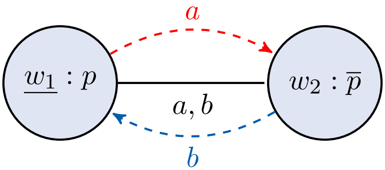

$\underline{w_1} : p$

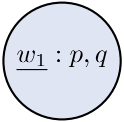

$\underline{w_1} : p$
$w_2 : \overline{p}$

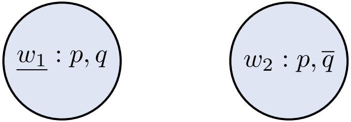

$\underline{w_1} : p$
a, b
$w_2 : \overline{p}$



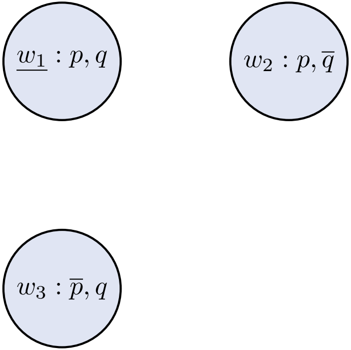
a
$\underline{w_1} : p$
$w_2 : \overline{p}$
a, b

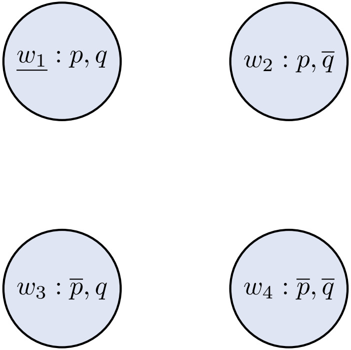

a
$\underline{w_1} : p$
$w_2 : \overline{p}$
a, b
b

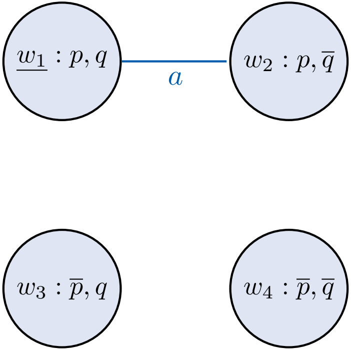

$\underline{w_1} : p, q$

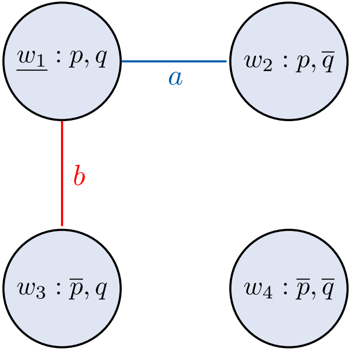


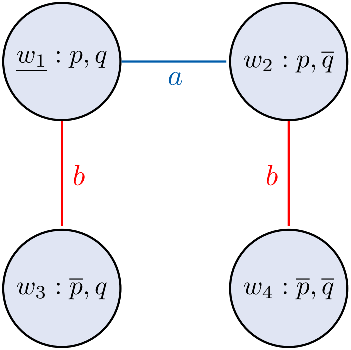
$\underline{w_1} : p, q$
$w_2 : p, \overline{q}$


$\underline{w_1} : p, q$
$w_2 : p, \overline{q}$
$w_3 : \overline{p}, q$

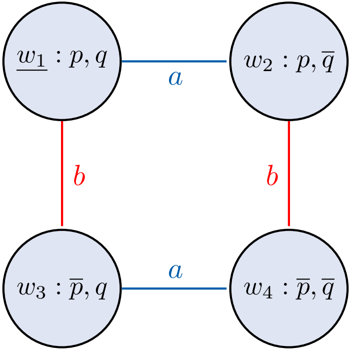

$\underline{w_1} : p, q$
$w_2 : p, \overline{q}$
$w_3 : \overline{p}, q$
$w_4 : \overline{p}, \overline{q}$


$\underline{w_1} : p, q$
a
$w_2 : p, \overline{q}$
$w_3 : \overline{p}, q$
$w_4 : \overline{p}, \overline{q}$

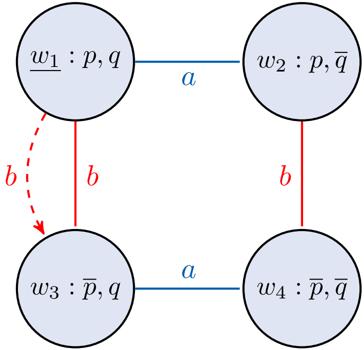

$\underline{w_1} : p, q$
a
$w_2 : p, \overline{q}$
b
$w_3 : \overline{p}, q$
$w_4 : \overline{p}, \overline{q}$



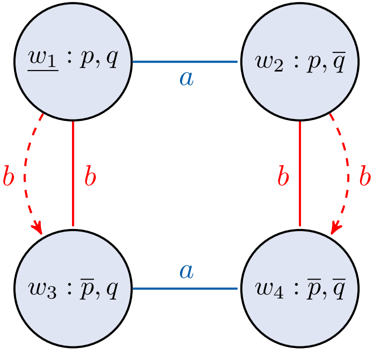
$\underline{w_1} : p, q$
$w_2 : p, \overline{q}$
a
b
b
$w_3 : \overline{p}, q$
$w_4 : \overline{p}, \overline{q}$

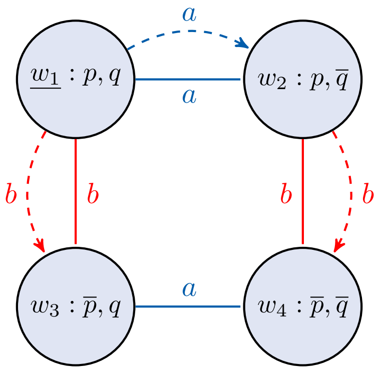


$\underline{w_1} : p, q$
$w_2 : p, \overline{q}$
$w_3 : \overline{p}, q$
$w_4 : \overline{p}, \overline{q}$
a
b
b
a

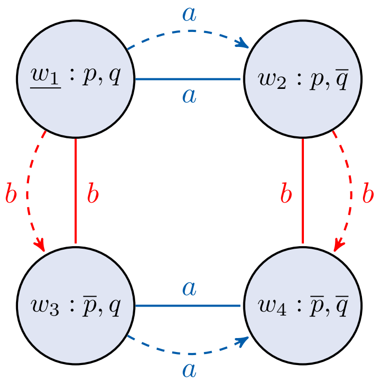


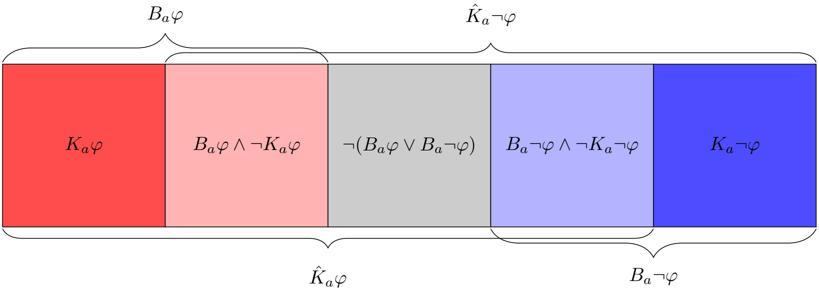
$\underline{w_1} : p, q$
$w_2 : p, \overline{q}$
$w_3 : \overline{p}, q$
$w_4 : \overline{p}, \overline{q}$
a
a
b
b
b


$\underline{w_1} : p, q$
$w_2 : p, \overline{q}$
$w_3 : \overline{p}, q$
$w_4 : \overline{p}, \overline{q}$
a
a
b
b
b
b



a
$\underline{w_1} : p, q$
$w_2 : p, \overline{q}$
a
b
b
b
b
$w_3 : \overline{p}, q$
a
$w_4 : \overline{p}, \overline{q}$


a
$\underline{w_1} : p, q$
$w_2 : p, \overline{q}$
a
b
b
b
b
$w_3 : \overline{p}, q$
a
$w_4 : \overline{p}, \overline{q}$
a


$B_a\varphi$
$\hat{K}_a\neg\varphi$
$K_a\varphi$
$B_a\varphi \wedge \neg K_a\varphi$
$\neg(B_a\varphi \vee B_a\neg\varphi)$
$B_a\neg\varphi \wedge \neg K_a\neg\varphi$
$K_a\neg\varphi$
$\hat{K}_a\varphi$
$B_a\neg\varphi$

STRONG COMMON BELIEF

Group Knowledge and Belief

$$\varphi := K_G\varphi \mid B_G\varphi \mid C_G\varphi \mid CB_G\varphi \mid CB^*_G\varphi$$

Everybody knows...

$$K_G\varphi := \bigwedge_{i \in G} K_i\varphi$$

Everybody knows (n degree)

$$K_G^n := \underbrace{K_G \dots K_G}_{n}$$

Common Knowledge

$$C_G\varphi := \bigwedge_{n=1}^{\infty} K_G^n\varphi = K_G\varphi \wedge K_G^2\varphi \wedge K_G^3\varphi \wedge \ldots$$

Properties of Common knowledge

- $\models C_G\varphi \to \varphi$
- $\models C_G\varphi \to C_G C_G\varphi$
- $\models \neg C_G\varphi \to C_G \neg C_G\varphi$

Everybody Believes...

- $B_G\varphi := \bigwedge_{i \in G} B_i\varphi$

Everybody believes (n degree)

- $B_G^n := \underbrace{B_G \ldots B_G}_{n}$

Common Belief

$$CB_G\varphi := \bigwedge_{n=1}^{\infty} B_G^n\varphi = B_G\varphi \wedge B_G^2\varphi \wedge B_G^3\varphi \wedge \ldots$$

Strong Common Belief

$$CB^*_G\varphi := C_G(B_G\varphi) = \bigwedge_{n=1}^{\infty} K^n_G(B_G\varphi)$$

Properties

1. $\models C_G\varphi \to C^*_B\varphi$
2. $\models C_G\varphi \to CB^*_G\varphi$
3. $\not\models C^*_B\varphi \to C_G\varphi$
4. $\models CB^*_G\varphi \to CB_G\varphi$
5. $\not\models CB_G\varphi \to CB^*_G\varphi$
6. $\not\models CB^*_G\varphi \to \varphi$
7. $\models CB^*_G\varphi \to CB^*_G CB^*_G\varphi$
8. $\models \neg CB^*_G\varphi \to CB^*_G \neg CB^*_G\varphi$

Property	Axiom	$C_G\varphi$	$CB_G\varphi$	$CB^*_G\varphi$
Factivity	$\Box\varphi \to \varphi$	✓	×	×
Positive Introspection	$\Box\varphi \to \Box\Box\varphi$	✓	✓	✓
Negative Introspection	$\neg\Box\varphi \to \Box\neg\Box\varphi$	✓	×	✓

Table: Properties of epistemic/doxatic operators

TAXONOMY OF ASSERTIONS

Literal assertion vs. Non-literal assertion

$S : \varphi$ in the context c

1. Non-literal assertion: $CB^*_{S,H} \neg \varphi$
2. Literal assertion: $\neg CB^*_{S,H} \neg \varphi$

1.Non-literal assertion

1. Non-literal assertion: $CB^*_{S,H}\neg\varphi$
   - 1.1. Conventional (non-literal) assertion
     $\exists\psi : CB^*_{S,H}(\varphi \leadsto \psi)$
     irony, metaphor, hyperbole etc.
   - 1.2. Nonconventional (non-literal) assertion
     $\neg\exists\psi : CB^*_{S,H}(\varphi \leadsto \psi)$

2. Literal assertion

2. Literal assertion: $\neg CB^*_{S,H}\neg\varphi$
   - 2.1. Trivial assertion
     $CB^*_{S,H}\varphi$
   - 2.2. Non-trivial assertion
     $\neg CB^*_{S,H}\varphi$

2. Literal> 2.1. Trivial assertion

2.1. Trivial assertion: $CB^*_{S,H}\varphi$

- 2.1.1. Conventional (trivial literal) assertion
  $\exists\psi : CB^*_{S,H}(\varphi \rightsquigarrow \psi)$
- 2.1.2. Non-conventional (trivial literal) assertion
  $\neg\exists\psi : CB^*_{S,H}(\varphi \rightsquigarrow \psi)$

2.2. Non-trivial assertion: $\neg CB^{*}_{S,H}\varphi$

- 2.2.1. Deceptive assertion
  $B_S \neg\varphi$
- 2.2.2. Non-deceptive assertion
  $\neg B_S \neg\varphi$

2. Literal > 2.2. Non-trivial > 2.2.2. Non-deceptive

2.2.2. Non-deceptive assertion: $\neg B_S \neg \varphi$

- 2.2.2.1. Truthful assertion
  $B_S \varphi$
- 2.2.2.2. Bluffing assertion
  $\neg B_S \varphi$

Epistemic Taxonomy of Assertions