Collective Obligations and Individualism

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Abstract

The individualist claim about collective obligations is that collective obligations are reducible to the individual obligations of the collective's members. This the collectivists deny. We propose to discover who is right by way of a deontic logic of collective action that models collective actions, abilities, obligations, and their interrelations. On the basis of our formal analysis, we argue that when assessing the obligations of an individual agent, we need to distinguish individual obligations from member obligations. If a collective has a collective obligation to bring about a particular state of affairs, then it might be that no individual in the collective has the individual obligation to bring about that state of affairs. What follows from a collective obligation is that each member of the collective has a member obligation to help ensure that the collective fulfills its collective obligation. In conclusion, we argue that our formal analysis supports collectivism.

Keywords: collective obligation, collective responsibility, individualism, collectivism, deontic logic, game theory.

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1 A Deontic Logic of Collective Action

1.1 Language

Our propositional modal language $\mathfrak{L}$ is built from a finite set $N = \{i_1, \ldots, i_n\}$ of individual agents and a countable set $\mathfrak{P} = \{p_1, p_2, \ldots\}$ of atomic formulas. The formal language $\mathfrak{L}$ is the smallest set (in terms of set-theoretical inclusion) that satisfies the conditions (i) through (iv):

(i) $\mathfrak{P} \subseteq \mathfrak{L}$
(ii) If $\phi \in \mathfrak{L}$, then $\neg\phi \in \mathfrak{L}$ and $\Diamond\phi \in \mathfrak{L}$
(iii) If $\phi \in \mathfrak{L}$ and $\psi \in \mathfrak{L}$, then $(\phi \wedge \psi) \in \mathfrak{L}$
(iv) If $\mathcal{G} \subseteq N$ and $\phi \in \mathfrak{L}$, then $[\mathcal{G}]\phi \in \mathfrak{L}$ and $(\mathcal{G})\phi \in \mathfrak{L}$.

We leave out brackets and braces if the omission does not give rise to ambiguities. The connectives $\vee$, $\rightarrow$, and $\Box$ abbreviate the usual constructions.

1.2 Semantics

Definition 1. *A* deontic game model M *is a quadruple* $\langle N, (A_i), d, v\rangle$, *where* N *is a finite set of individual agents, for each agent* i *in* N *it holds that* A_i *is a non-empty and finite set* A_i *of actions available to agent* i, d *is a deontic ideality function that assigns to each outcome* a *in* A *(*$= \times_{i\in N} A_i$*) a value* $d(a) \in \{0, 1\}$*, and* v *a valuation function that assigns to each atomic formula* p *a set of outcomes* $v(p) \subseteq A$ *(*$= \times_{i\in N} A_i$*).*

Definition 2 (Dominance)**.** *Let* $M = \langle N, (A_i), d, v\rangle$ *be a deontic game model. Let* $\mathcal{G} \subseteq N$ *be a group of agents. Let* $a_\mathcal{G}, a'_\mathcal{G} \in A_\mathcal{G}$ *be collective actions available to group* $\mathcal{G}$*. Then*

$$a_\mathcal{G} \succeq a'_\mathcal{G} \quad \textit{iff} \quad \textit{for all } a_{-\mathcal{G}} \in A_{-\mathcal{G}} \textit{ it holds that } d(a_\mathcal{G}, a_{-\mathcal{G}}) \geq d(a'_\mathcal{G}, a_{-\mathcal{G}}).$$

Strong dominance is defined in terms of weak dominance: $a_\mathcal{G} \succ a'_\mathcal{G}$ if and only if $a_\mathcal{G} \succeq a'_\mathcal{G}$ and $a'_\mathcal{G} \not\succeq a_\mathcal{G}$.

Definition 3 (Optimality)**.** *Let* $M = \langle N, (A_i), d, v\rangle$ *be a deontic game model. Let* $\mathcal{G} \subseteq N$ *be a group of agents. Then the set of* $\mathcal{G}$*'s* optimal actions*, denoted by Optimal(*$\mathcal{G}$*), is given by*

$$\textit{Optimal}(\mathcal{G}) \quad = \quad \{a'_\mathcal{G} \in A_\mathcal{G} : \textit{there is no } a_\mathcal{G} \in A_\mathcal{G} \textit{ such that } a_\mathcal{G} \succ a'_\mathcal{G}\}.$$

Definition 4 (Semantical Rules)**.** *Let* $M = \langle N, (A_i), d, v \rangle$ *be a deontic game model. Let* $\mathcal{G} \subseteq N$ *be a group of agents. Let* $a, a' \in A$ *be outcomes. Let* $p \in \mathfrak{P}$ *be an atomic formula and* $\phi, \psi \in \mathfrak{L}$ *be arbitrary formulas. Then*

$M, a \models p$	*iff*	$a \in v(p)$
$M, a \models \neg\phi$	*iff*	$M, a \not\models \phi$
$M, a \models \phi \wedge \psi$	*iff*	$M, a \models \phi$ *and* $M, a \models \psi$
$M, a \models \Diamond\phi$	*iff*	*there is an* a' *such that* $M, a' \models \phi$
$M, a \models [\mathcal{G}]\phi$	*iff*	*for all* a' *with* $a'_{\mathcal{G}} = a_{\mathcal{G}}$ *it holds that* $M, a' \models \phi$
$M, a \models (\mathcal{G})\phi$	*iff*	*for all* a' *with* $a'_{\mathcal{G}} \in$ *Optimal*$(\mathcal{G})$ *it holds that* $M, a' \models \phi$.

Lemma 1. *Let* $\phi, \psi \in \mathfrak{L}$ *and* $\mathcal{G} \subseteq N$. *Then*

$$\models (\mathcal{G})\phi \to \Diamond[\mathcal{G}]\phi$$
$$\models [\mathcal{G}](\phi \wedge \psi) \leftrightarrow ([\mathcal{G}]\phi \wedge [\mathcal{G}]\psi)$$
$$\models (\mathcal{G})(\phi \wedge \psi) \leftrightarrow ((\mathcal{G})\phi \wedge (\mathcal{G})\psi).$$

Lemma 2. *Let* $\phi \in \mathfrak{L}$ *and* $\mathcal{F} \subseteq \mathcal{G} \subseteq N$. *Then*

$$\not\models (\mathcal{F})\phi \to (\mathcal{G})\phi$$
$$\not\models (\mathcal{G})\phi \to (\mathcal{F})\phi$$
$$\models (\mathcal{F})\phi \to \neg(\mathcal{G})\neg\phi$$
$$\models (\mathcal{G})\phi \to \neg(\mathcal{F})\neg\phi.$$

Lemma 3. *Let* $\phi \in \mathfrak{L}$ *and* $\mathcal{F}_1, \mathcal{F}_2 \subseteq \mathcal{G} \subseteq N$. *Then*

$$\not\models \neg((\mathcal{F}_1)\phi \wedge (\mathcal{F}_2)\neg\phi)$$
$$\models ((\mathcal{F}_1)\phi \wedge (\mathcal{F}_2)\neg\phi) \to (\neg(\mathcal{G})\phi \wedge \neg(\mathcal{G})\neg\phi)$$
$$\models ((\mathcal{G})\phi \vee (\mathcal{G})\neg\phi) \to \neg((\mathcal{F}_1)\phi \wedge (\mathcal{F}_2)\neg\phi).$$